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GENERAL THEOREMS OF THE ELECTROMECHANICS OF THIN ELASTIC SHELLS*

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The properties of the problem of the dynamics of thin elastic electrically conducting shells in arbitrary electromagnetic fields are considered. On the basis of the non-linear equations /1/ obtained by asymptotic integration of the equations of elasticity theory and Maxwell equations over the thickness (in the quasistationary approximation) an expression is derived for the functional of the power governing the power balance in the process being investigated. A complete system of natural boundary conditions is formulated for the problem. Orthogonality conditions for the eigensolutions expressing the appropriate formulations of the reciprocity theorem are deduced for two fundamental problems of the electromechanics of thin elastic shells /1/ whose equations are linear and can be obtained from the original equations without loss of accuracy. The question of the decomposition of solutions of the inhomogeneous problems that occur here in eigensolutions is considered.

Questions of the energy balance and uniqueness of the solutions in the first linear problem (magnetoelasticity) were also examined in /2, 3/.

1. We will consider a triorthogonal system of coordinates $(\alpha_1, \alpha_2, \alpha_3)$, given in an infinite space V around a shell in which the shell can be considered as a mathematical slit over its middle surface S. Let S lie on the coordinate surface $\alpha_3 = 0$ and be either closed or bounded by a closed line G coincident with the shell edges. We identify the properties of the surrounding medium with the properties of a vacuum. We assume the shell to be elastic, non-magnetic, and to have finite electrical conductivity.

The solution of the problem of small forced electrical vibrations of such a shell in an arbitrary, generally time-varying electromagnetic field is the simultaneous integration of the equations

$$\begin{gathered} 2EhL\mathbf{u} - \mathbf{X}^{(i)} - \mathbf{X}^{(p)} - \mathbf{X}^{(m)} = 0 \quad \text{on } S \\ \gamma\Delta_s F + B_3^{\cdot} - \operatorname{rot}_3(\mathbf{u}^{\cdot} \times \mathbf{B}) = 0 \quad \text{on } S \\ \Delta\Phi = 0 \text{ in } V \end{gathered} \tag{1.1}$$

with satisfaction the condition

$$(\partial\Phi/\partial\alpha_3)_s^+ = (\partial\Phi/\partial\alpha_3)_s^- \tag{1.2}$$

Here

$$\mathbf{X}^{(i)} = -2\rho h\mathbf{u}^{\cdot\cdot}, \quad \mathbf{X}^{(p)} = -\mu_1^{-1}(\operatorname{grad}_s F \times \mathbf{i}_3) \times \mathbf{B} \tag{1.3}$$

and vectors of the inertial forces $(\mathbf{X}^{(i)})$ and the ponderomotive forces $(\mathbf{X}^{(p)})$, and $\mathbf{X}^{(m)}$ is the vector of active forces of mechanical origin;

$$\begin{gathered} \mathbf{B} = \mathbf{B}^{\circ} + \mathbf{f}, \quad \mathbf{f} = {}^1/_2[(\operatorname{grad}\Phi)_s^+ + (\operatorname{grad}\Phi)_s^-] \\ F = (\Phi)_s^+ - (\Phi)_s^- \end{gathered} \tag{1.4}$$

are formulas connecting the values of the vectors on the surface S: the total magnetic induction $(\mathbf{B})$, the given magnetic induction of the secondary sources $(\mathbf{B}^{\circ})$ and the magnetic induction of the eddy currents $(\mathbf{f})$, Φ is the potential of the eddy current magnetic induction $\mathbf{b}$ (determined in V by the expression $\mathbf{b} = \operatorname{grad}\Phi$), F is the drop in the potential Φ on S, $\mathbf{u}$ is the vector of elastic displacements of the shell middle surface, L is the operator of the shell theory equations in displacements /4/, their expressions in the domains V and S, respectively are taken for the Laplace (Δ) and gradient operators, without and with the subscript s, the following notation is used $(\cdot)_s^{\pm} = (\cdot)_{\alpha_3\to\pm 0}$, $\operatorname{rot}_3(\cdot) = [\operatorname{rot}(\cdot)\cdot\mathbf{i}_3]_s$, $\mathbf{i}_1 - \mathbf{i}_3$ are

*Prikl. Matem. Mekhan., 53, 4, 658-664, 1989 unit direction along $\alpha_1 - \alpha_3$, $\gamma = (2h\mu_0\sigma)^{-1}$; $\rho, E, 2h, \sigma$ are the density, Young's modulus, and the shell thickness and conductivity, μ_0 is the magnetic constant and the dot denotes the derivative with respect to time t.

For values of $\mathbf{B}^\circ$ and $\mathbf{X}^{(m)}$ given as functions of time and the coordinates, (1.1)-(1.4) form a closed non-linear system in the unknowns $\mathbf{u}$ and Φ to which certain boundary conditions at the edge G and at infinity /5/ must be appended for the solution. They are discussed below in Sect.3.

Eqs.(1.1)-(1.4) are the vector mode of writing equations /1/ obtained by the asymptotic method by integrating the equations of elasticity theory and Maxwell equations over the shell thickness within the framework of certain assumptions, particularly the boundedness of the variability of the processes under investigation in time by those limits in which no skin-effect will appear in the shell thickness /6/ and the two-dimensional dynamic theory of elastic shells holds /4/.

The linear eddy current density $\mathbf{J}$ and the electric field $\mathbf{e}$ in the shell can be expressed in terms of the quantities introduced by means of the equalities

$$\mathbf{J} = \mu_0^{-1}\,\mathrm{grad}_s F \times \mathbf{i}_3, \quad \mathbf{e} = (2h\sigma)^{-1}\mathbf{J} - \mathbf{u}^\cdot \times \mathbf{B} \tag{1.5}$$

2. Let $(\mathbf{u}, \Phi)$ be a certain solution of Eqs.(1.1)-(1.4). Then the power balance in the problem determined by these equations is expressed by the functional

$$N = N^{(m)} + N^{(i)} + N_s^{(e)} + N_v^{(e)} - P_g^{(m)} - P_g^{(e)} - P_\infty^{(e)} - P_s^{(m)} - P_s^{(e)} = 0 \tag{2.1}$$

where $N^{(m)}$ is the elastic strain power of the shell, $N^{(i)}$ is the power of the shell inertial forces, $N_s^{(e)}$ is the eddy current power in the shell, $N_v^{(e)}$ is the power of the induced electromagnetic field in the space surrounding the shell, $P_g^{(m)}$ is the power of the edge forces and moments in the shell, $P_g^{(e)}$ is the edge current power, $P_\infty^{(e)}$ is the power loss by radiation, and $P_s^{(m)}$ and $P_s^{(e)}$ are, respectively, the active power of the dynamic load of mechanical and electromagnetic origin. In the general case they are expressed by the formulas

$$\begin{gathered}
N^{(m)} = \iint (T_{jk}\varepsilon^{jk\cdot} + M_{jk}\mu^{jk\cdot})\,ds, \quad N^{(i)} = \rho h\Big[\iint \mathbf{u}^{\cdot 2} ds\Big]^\cdot \\
N_s^{(e)} = \lambda\mu_0^{-1}\iint (\mathrm{grad}_s F)^2\,ds, \quad N_v^{(e)} = (2\mu_0)^{-1}\Big[\iiint (\mathrm{grad}\,\Phi)^2\,dv\Big]^\cdot \\
P_g^{(m)} = \oint (\mathbf{P}_k\mathbf{u}^\cdot + \mathbf{Q}_k\mathbf{\Gamma}^\cdot)A_k\,d\alpha_k, \quad P_g^{(e)} = \mu_0^{-1}\oint F e_k A_k\,d\alpha_k \\
P_\infty^{(e)} = \mu_0^{-1}\iint \Phi\Big(\frac{\partial\Phi^\cdot}{\partial N}\Big)d\sigma \\
P_s^{(m)} \iint \mathbf{X}^{(m)}\cdot\mathbf{u}^\cdot ds, \quad P_s^{(e)} = \mu_0^{-1}\iint B_3^{\circ\cdot} F\,ds \\
ds = A_1A_2\,d\alpha_1 d\alpha_2, \quad dv = A_1A_2A_3\,d\alpha_1 d\alpha_2 d\alpha_3
\end{gathered} \tag{2.2}$$

Here $A_1 - A_3$ are coefficients of the first quadratic form of the given coordinate system, $d\sigma$ is an element of the surface Σ enclosing the shell (by taking the latter to be a sphere of radius r we will assume that $r \to \infty$), $\mathbf{N}$ is the unit normal to Σ external to V (and later to S) T_{jk}, M_{jk}, ε_{jk}, μ_{jk} are tensors of the forces, moments, strains, and angles of rotation of the shell, $\mathbf{P}, \mathbf{Q}, \mathbf{\Gamma}$ are the boundary values of the vectors of the surface forces, moments, and elastic rotations, and $j, k = 1, 2$ are the subscripts over which summation is made. Integration, here and later is over the surfaces S, Σ, the volume V, and the contour G with elements ds, $d\sigma$, dv, $d\alpha_k$ unless otherwise stated.

Computations resulting in obtaining the relationships (2.1) and (2.2) can be reduced to the following scheme. We write the first two equations in (1.1) symbolically in the form $\mathbf{M}(\mathbf{u}, \Phi, \mathbf{X}^{(m)}) = 0$ and $E(\mathbf{u}, \Phi, B_3^\circ) = 0$, respectively, and we differentiate the last equation in (1.1) with respect to t. Multiplying these equations by a certain quantity and integrating over the domains of definition, we form the sum

$$\mathbf{N} = \iint \Big[\mathbf{M}(\mathbf{u}, \Phi, \mathbf{X}^{(m)})\mathbf{u}^\cdot - E(\mathbf{u}, \Phi, B_3^\circ)\frac{F}{\mu_0}\Big]ds - \iiint \Big[\Delta(\Phi^\cdot)\frac{\Phi}{\mu_0}\Big]dv = 0 \tag{2.3}$$

Let us expand the expressions in (2.3)

$$\iiint 2Eh\,(L\mathbf{u})\,\mathbf{u}^\cdot\,ds = N^{(m)} - P_g^{(m)} \tag{2.4}$$

Eq.(2.4) follows directly from the results of /4/ if the operator L is formally self-adjoint. The power of the elastic strains $N^{(m)}$ can here be expressed in terms of the shell strain potential energy W by means of the formula $N^{(m)} = W^\cdot$ where

$$W = \frac{2Eh}{1-\nu^2}\iint \Big\{\Big[\varepsilon_1{}^2 + \varepsilon_2{}^2 + 2\nu\varepsilon_1\varepsilon_2 + \frac{1-\nu}{2}\,\omega^2\Big] + \frac{h^2}{3}\,[\varkappa_1{}^2 + \varkappa_2{}^2 + 2\nu\varkappa_1\varkappa_2 + (1-\nu)\,\tau^2]\Big\}\,ds$$

ε_k, ω, $\varkappa_k$, τ are components of the tangential and bending strains of the shell and ν is Poisson's ratio. The equalities

$$-\iint \mathbf{X}^{(i)}\mathbf{u}^{\cdot}\,ds = N^{(i)}, \quad -\iint \Big(\frac{B_3{}^{\cdot}F}{\mu_0}\Big)\,ds = -P_s^{(e)} - \mu_0^{-1}\iint f_3{}^{\cdot}F\,ds \tag{2.5}$$

can be confirmed by direct substitution taking (1.3) and (1.4) into account.

Using (1.3) and applying Stokes's theorem we have

$$\iint\Big[-\mathbf{X}^{(p)}\mathbf{u}^{\cdot} + \mathrm{rot}_3\,(\mathbf{u}^{\cdot}\times\mathbf{B})\,\frac{F}{\mu_0}\Big]\,ds = \mu_0^{-1}\iint \mathrm{rot}_3\,[F\,(\mathbf{u}^{\cdot}\times\mathbf{B})]\,ds = -\mu_0^{-1}\oint F\,(-1)^k\,(B_3 u_l{}^{\cdot} - B_l u_3{}^{\cdot})\,A_k\,d\alpha_k, \quad l, k = 1, 2, \quad l \neq k \tag{2.6}$$

Green's theorem is used in two cases. We hence obtain

$$-\iint\Big[\gamma\Delta_s\,(F)\,\frac{F}{\mu_0}\Big]\,ds = N_s^{(e)} - \gamma\mu_0^{-1}\oint F\,(-1)^k\,A_l^{-1}\,\frac{\partial F}{\partial\alpha_l}\,A_k d\alpha_k \tag{2.7}$$

$$-\iiint\Big[\Delta(\Phi^{\cdot})\,\frac{\Phi}{\mu_0}\Big]\,dv = N_v^{(e)} - \mu_0^{-1}\iint \Phi\Big(\frac{\partial\Phi^{\cdot}}{\partial N}\Big)\,d\sigma - \mu_0^{-1}\iint_{S^+ + S^-}\Phi\Big(\frac{\partial\Phi^{\cdot}}{\partial N}\Big)\,ds = N_v^{(e)} - P_\infty^{(e)} + \mu_0^{-1}\iint f_3{}^{\cdot}F\,ds \tag{2.8}$$

Integration in the surface integral in (2.8) is over the whole surface $S^+ + S^- + \Sigma$, enclosing V where $S^\pm$ are understood to be the facial surfaces $\alpha_3 = \pm 0$ on a mathematical slit S. It is taken into account in obtaining the last component (2.8) that $(\partial\Phi/\partial N)_s{}^+ = -(\partial\Phi/\partial\alpha_3)_s{}^+$, $(\partial\Phi/\partial N)_s{}^- = (\partial\Phi/\partial\alpha_3)_s{}^-$, and also the equalities (1.2) and (1.4).

Summing the expressions in (2.3), taking (2.4)-(2.8) into account, we obtain (2.1). We note that the last components in (2.5) and (2.8) cancel one another here while (2.6) and the last component in (2.7) yield $P_g{}^{(e)}$ when added (the second formula in (1.7) must also be taken into account here).

We assume that the domain V can contain non-variable closed subdomains (V', V'') that do not make contact with the facial surfaces S and are occupied by material with the properties of an ideal dielectric (V') or an ideal conductor (V'') (the subdomains V'' should be simply-connected). An ideal dielectric does not distort the field Φ while the normal component $\mathbf{b}$ ($\mathbf{n}'$ is the normal) on the surface of an ideal conductor equals zero and therefore $\partial\Phi^{\cdot}/\partial n' = 0$ in the surface integral (2.8). Consequently, the balance of the powers (2.1) and (2.2) remains valid. We will also assume that the condition $\mathbf{u}\cdot\mathbf{n}' = 0$ is satisfied on sections of the boundary G in contact with the ideal conductor, excepting penetration of the boundary into V''.

3. We will examine the boundary conditions that must be appended to Eqs.(1.1)-(1.4) by limiting the examination to those for which $P_g{}^{(m)} + P_g{}^{(e)} + P_\infty{}^{(e)} = 0$ for compliance.

The first group of conditions in mechanical in nature, forms an ordinary system of shell theory boundary conditions and, as is shown in /5/, ensures that $P_g{}^{(m)}$ will equal zero for such idealized boundary conditions as rigid clamping, hinge support, and a free edge, say.

The second group of conditions consists of satisfying the ordinary condition of boundedness /6/ of the potential Φ at infinity (here $P_\infty{}^{(e)} = 0$), the condition $\partial\Phi/\partial n' = 0$ on the surface of the subdomain V'' and one of the following conditions ($\boldsymbol{\tau}$ and $\mathbf{n}$ are the tangent and normal to G):

$$F = 0 \ \text{ or } \ \partial F/\partial n = 0 \text{ on } G \tag{3.1}$$

Their physical meaning is as follows. As follows from (1.5), the first and second conditions in (3.1) denote that the normal and tangential components of the linear eddy current density to G equal zero, which corresponds to conditions of a boundary insulated from and in contact with an ideal conductor. Taking into account that in the case of boundary contact with an ideal conductor $\mathbf{u}\cdot\mathbf{n}' = 0$ (see Sect.2) and $\mathbf{B}\cdot\mathbf{n}' = 0$, it is easy to obtain the equality $e_\tau = (2h\sigma)^{-1}\partial F/\partial n$ from (1.5). Therefore, satisfaction of Conditions (3.1) ensures the equality $P_g{}^{(e)} = 0$.

4. Eqs.(1.1)-(1.4) are non-linear. They can be linearized /1/ without loss of accuracy when solving the following two fundamental problems of the electromechanics of thin elastic shells.

Problem 1 is to determine the influence of a constant (in time) magnetic field $\mathbf{B}^\circ$ on the free vibrations $(\mathbf{X}^{(m)} = 0)$ or forced vibrations $(\mathbf{X}^{(m)} = \mathbf{X}^{(m)}(t))$ of a shell. The appropriate linear form of the equations can be obtained from (1.1)-(1.4) if we put $B_3^{\cdot} = f_3^{\cdot}$ in the second equation in (1.1), while it is assumed that $\mathbf{B} = \mathbf{B}^\circ$ in the remaining equations. These simplifications are associated with the fact that the magnetic field occurring because of small shell vibrations is small compared with the magnetic field of the secondary sources.

Problem 2 is to determine the elastic vibrations of a shell caused by a variable magnetic field $\mathbf{B}^\circ = \mathbf{B}^\circ(t)$. The appropriate equations can be obtained from (1.1)-(1.4) by neglecting the last component $\operatorname{rot}_3(\mathbf{u}^{\cdot} \times \mathbf{B})$ in the second equation in (1.1). This is related to the fact that the eddy currents caused by small shell vibrations are small compared with the currents induced by the variable magnetic field $\mathbf{B}^\circ$ of the secondary sources.

The solution of Problem 1 requires the joint integration of the appropriate equations.

The equations of Problem 2 split into two subsystems whose integration determines the two succeeding steps in solving the problem. The first step is the joint integration of the equations obtained from the second and third equations in (1.1). The field Φ (and the function F) is hence determined. The second step is to solve the usual problem of integrating the equations of shell motion (the first equation in (1.1)) for a known pressure $\mathbf{X}^{(p)}$ (determined by direct actions from (1.2)) and the pressure of the active forces $\mathbf{X}^{(m)}$ (these may not be present, $\mathbf{X}^{(m)} = 0$). The integration problems solved in both steps are linear.

Problem 1 agrees with the problem of the magnetoelasticity of thin shells formulated in /7/, while the equations of Problem 1, as shown in /8/, can be obtained from the equations in /7/ by certain manipulations including the introduction of a magnetic potential and discarding asymptotically small terms. The first step in Problem 2 is identical with the electrodynamic problems of thin conducting shells /9/ while the appropriate equations can be reduced to the equations in /10/.

5. Let us examine Problem 1 by assuming that the perturbing forces $\mathbf{X}^{(m)}$ and all the quantities desired vary with time as $\exp(\Omega t)$. After discarding this factor in the equations of Problem 1 we obtain them in the form

$$\mathbf{M}_1(\mathbf{u}, \Phi, \Omega, \mathbf{X}^{(m)}) = 2EhL\mathbf{u} + 2\rho h\Omega^2\mathbf{u} + \mu_0^{-1}(\operatorname{grad}_s F \times \mathbf{i}_3) \times \mathbf{B}^\circ - \mathbf{X}^{(m)} = 0 \tag{5.1}$$

$$E_1(\mathbf{u}, \Phi, \Omega) = \gamma\Delta_s F + \Omega f_3 - \Omega \operatorname{rot}_3(\mathbf{u} \times \mathbf{B}^\circ) = 0, \quad \Delta\Phi = 0$$

We will first investigate the properties of the eigensolutions of Problem (5.1) by setting $\mathbf{X}^{(m)} = 0$.

Let $(\mathbf{u}_{(t)}, \Phi_{(t)}, \Omega_{(t)})$ and $(\mathbf{u}_{(q)}, \Phi_{(q)}, \Omega_{(q)})$ be two eigensolutions of the homogeneous Eqs. (5.1) that satisfy the boundary conditions listed in Sect.3 that ensure that $P_g^{(m)} + P_g^{(e)} + P_\infty^{(e)}$ equals zero. (It is henceforth assumed everywhere that the solutions of all the boundary value problems under consideration satisfy such boundary conditions).

We form the functional

$$N_{(tq)} = \iint \left[\mathbf{M}_1(\mathbf{u}_{(t)}, \Phi_{(t)}, \Omega_{(t)}, 0)\,\Omega_{(q)}\mathbf{u}_{(q)} - E_1(\mathbf{u}_{(q)}, \Phi_{(q)}, \Omega_{(q)})\frac{F_{(t)}}{\mu_0}\right] ds - \iiint \left[\Omega_{(q)}\Delta(\Phi_{(q)})\frac{\Phi_{(t)}}{\mu_0}\right] dv = 0, \quad F_{(t)} = (\Phi_{(t)})_s^+ - (\Phi_{(t)})_s^-$$

Using the procedures described in Sect.2, we obtain

$$N_{(tq)} = \Omega_q W_{(tq)} + \Omega_{(t)}^2\Omega_{(q)} I_{(tq)} + P_{(tq)} + \Omega_{(q)} Q_{(tq)} = 0 \tag{5.2}$$

$$W_{(tq)} = \frac{2Eh}{1-\nu^2}\iint \Big\{\Big[\varepsilon_1^{(t)}\varepsilon_1^{(q)} + \varepsilon_2^{(t)}\varepsilon_2^{(q)} + \nu(\varepsilon_1^{(t)}\varepsilon_2^{(q)} + \varepsilon_1^{(q)}\varepsilon_2^{(t)}) + \frac{1-\nu}{2}\omega^{(t)}\omega^{(q)}\Big] + {}^1/_3 h^2[\varkappa_1^{(t)}\varkappa_1^{(q)} + \varkappa_2^{(t)}\varkappa_2^{(q)} + \nu(\varkappa_1^{(t)}\varkappa_2^{(q)} + \varkappa_1^{(q)}\varkappa_2^{(t)}) + (1-\nu)\tau^{(t)}\tau^{(q)}]\Big\} ds$$

$$I_{(tq)} = 2\rho h\iint \mathbf{u}_{(t)}\mathbf{u}_{(q)}\, ds, \quad P_{(tq)} = \gamma\mu_0^{-1}\iint \operatorname{grad}_s F_{(\,)} \operatorname{grad}_s F_{(q)}\, ds$$

$$Q_{(tq)} = \mu_0^{-1}\iiint \operatorname{grad}\Phi_{(t)} \operatorname{grad}\Phi_{(q)}\, dv$$

Taking account of the symmetry of the integrands $W_{(tq)}, I_{(tq)}, P_{(tq)}, Q_{(tq)}$ with respect to the subscripts (t, q), we form the difference $N_{(tq)} - N_{(qt)}$ and neglecting the common factor $(\Omega_{(q)} - \Omega_{(t)})$ we obtain the equality

$$W_{(tq)} + Q_{(tq)} - \Omega_{(t)}\Omega_{(q)} I_{(tq)} = 0, \quad t \neq q \tag{5.3}$$

Forming the differences $\Omega_{(t)} N_{(tq)} - \Omega_{(q)} N_{(qt)}$ and $\Omega_{(q)} N_{(tq)} - \Omega_{(t)} N_{(tq)}$, we can obtain two more equations analogous to (5.3)

$$P_{(tq)} + \Omega_{(t)}\Omega_{(q)} (\Omega_{(t)} + \Omega_{(q)}) I_{(tq)} = 0, \quad P_{(tq)} + (\Omega_{(t)} + \Omega_{(q)}) (W_{(tq)} + Q_{(tq)}) = 0 \tag{5.4}$$

Eqs.(5.3) and (5.4) are three different kinds of analogue for the conditions of orthogonality of the eigensolutions in the problem of the magnetoelasticity of thin shells.

We will now examine the problem of expanding the forms of forced vibrations of a shell under the action of a harmonic force $\mathbf{X}^{(m)}$ acting at the angular frequency Ω_*, in eigensolutions of the problem. To do this we set $\Omega = i\Omega_*$, in (5.1) and we seek the solution in the form

$$\mathbf{u} = \sum_{n=1}^{\infty} A_n \mathbf{u}_{(n)}, \quad \Phi = \sum_{n=1}^{\infty} A_n \Phi_{(n)} \tag{5.5}$$

We form the functional

$$\iint \left[\mathbf{M}_1 (\mathbf{u}, \Phi, i\Omega_*, \mathbf{X}^{(m)}) \Omega_{(q)} \mathbf{u}_{(q)} - E_1 (\mathbf{u}_{(q)}, \Phi_{(q)}, \Omega_{(q)}) \frac{F}{\mu_0} \right] ds -$$
$$\iiint \left[\Omega_{(q)} \Delta (\Phi_{(q)}) \frac{\Phi}{\mu_0} \right] dv = 0, \quad F = \sum_{n=1}^{\infty} A_n \Gamma_{(n)}$$

from which we obtain the equality

$$\sum_{n=1}^{\infty} (\Omega_*^2 + \Omega_{(n)}^2) I_{(nq)} A_n = - \iint \mathbf{X}^{(m)} \mathbf{u}_{(q)} ds$$

by taking account of (5.5), (5.2) and (5.3).

Specifying the values $1, 2, \ldots$ to q we obtain an infinite system of linear equations in the coefficients A_n of the expansions (5.5).

6. We will examine the first step in the solution of Problem 2 by assuming that $\mathbf{B}^\circ$ and Φ vary with time as $\exp(\Omega t)$. We here start from the equations

$$E_2 (\Phi, \Omega, B_3^\circ) = \gamma \Delta_s F + \Omega f_3 + \Omega B_3^\circ = 0, \ \Delta\Phi = 0 \tag{6.1}$$

Let $(\Phi_{(t)}, \Omega_{(t)})$ and $(\Phi_{(q)}, \Omega_{(q)})$ be two eigensolutions of the homogeneous Eqs.(6.1) $(B_3^\circ = 0)$

We form the functional

$$N_{(tq)} = - \iint \left[E_2 (\Phi_{(t)}, \Omega_{(t)}, 0) \frac{F_{(q)}}{\mu_0} \right] ds - \iiint \left[\Omega_{(t)} \Delta (\Phi_{(t)}) \frac{\Phi_{(q)}}{\mu_0} \right] dv =$$
$$P_{(tq)} - \Omega_{(t)} Q_{(tq)} = 0 \tag{6.2}$$

Taking the difference $N_{(tq)} - N_{(qt)}$ and discarding the common factor $(\Omega_{(t)} - \Omega_{(q)})$, using (6.2) we obtain two equations

$$Q_{(tq)} = 0, \quad P_{(tq)} = 0, \quad t \neq q \tag{6.3}$$

that express the orthogonality conditions and reciprocity theorem in the electrodynamic problem of thin conducting shells.

The eddy currents and the electromagnetic field for a shell in a harmonic field $\mathbf{B}^\circ$ (of angular frequency Ω_*) of secondary sources, will be determined by eigenfunction expansions by setting $\Omega = i\Omega_*$, in (6.1) and we represent Φ in the form of the sum (5.5).

Let us form the functional

$$\iint \left[E_2 (\Phi, i\Omega_*, B_3^\circ) \frac{F_{(q)}}{\mu_0} \right] ds + \iiint \left[i\Omega_* \Delta (\Phi) \frac{\Phi_{(q)}}{\mu_0} \right] dv = 0$$

from which we obtain an expression for the coefficients A_n of the expansion (5.5) for Φ by taking account of (6.2)

$$A_n = \frac{i\Omega_*}{i\Omega_* - \Omega_{(n)}} \iint B_3{}^\circ F_{(n)} \frac{ds}{(\mu_0 Q_{(nn)})}$$

According to (6.2), the substitution $Q_{(nn)} = -P_{(nn)}/\Omega_{(n)}$ can be made here.

The equations and properties of the solutions of the second step in Problem 2 are investigated in detail in shell theory. It must here be kept in mind that because of the non-linearity of (1.3) for $\mathbf{X}^{(p)}$ in the derivatives of the function Φ the magnetic pressure on the shell contains a constant (time-independent) component and a harmonic component (with angular frequency $2\Omega_*$).

7. As is seen from (2.1) and (2.2) the electromechanical process in a shell is non-conservative and accompanied by a power loss governed by the component $N_s^{(e)}$ and related to the liberation of heat because of the heating of the shell by the eddy currents. Consequently, the problem should generally be considered from the aspect of magnetothermoelasticity /11/ i.e., temperature stresses should be taken into account. Leaving this question as outside the scope of this paper, we note that thermal losses in a shell occur in the component $N_s^{(e)}$ of (2.1), while they can be determined (taking (6.3) into account) from the formula

$$\tfrac{1}{2}\gamma\mu_0^{-1} \iint \operatorname{grad}_s F \, \overline{\operatorname{grad}_s F} \, ds$$

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